Practice Problems

1. A concave mirror has a radius of curvature of $0.8\ \mathrm{m}$. What is the focal length of the mirror?

a. $-0.8\ \mathrm{m}$
b. $-0.4\ \mathrm{m}$
c. $0.4\ \mathrm{m}$
d. $0.8\ \mathrm{m}$

2. What is the focal length of a makeup mirror that produces a magnification of 1.50 when a person's face is 12.0 cm away? Construct a ray diagram using paper, a pencil and a ruler to confirm your calculation.

a. −36.0 cm
b. −7.20 cm
c. 7.20 cm
d. 36.0 cm

Check Your Understanding

3. How does the object distance, d_o, compare with the focal length, f, for a concave mirror that produces an image that is real and inverted?

a. $d_o > f$, where d_o and f are object distance and focal length, respectively.
b. $d_o < f$, where d_o and f are object distance and focal length, respectively.
c. $d_o = f$, where do and f are object distance and focal length, respectively.
d. $d_o = 0$, where do is the object distance.

4. Use the law of reflection to explain why it is not a good idea to polish a mirror with sandpaper.

a. The surface becomes smooth, and a smooth surface produces a sharp image.
b. The surface becomes irregular, and an irregular surface produces a sharp image.
c. The surface becomes smooth, and a smooth surface transmits light, but does not reflect it.
d. The surface becomes irregular, and an irregular surface produces a blurred image.

5. An object is placed in front of a concave mirror at a distance that is greater than the focal length of the mirror. Will the image produced by the mirror be real or virtual? Will it be erect or inverted?

a. It is real and erect.
b. It is real and inverted.
c. It is virtual and inverted.
d. It is virtual and erect.

16.2 Refraction

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain refraction at media boundaries, predict the path of light after passing through a boundary (Snell's law), describe the index of refraction of materials, explain total internal reflection, and describe applications of refraction and total internal reflection
- Perform calculations based on the law of refraction, Snell's law, and the conditions for total internal reflection

Section Key Terms

angle of refraction	corner reflector	critical angle	dispersion	incident ray
index of refraction	refracted ray	Snell's law	total internal reflection	

The Law of Refraction

You may have noticed some odd optical phenomena when looking into a fish tank. For example, you may see the same fish appear to be in two different places (Figure 16.16). This is because light coming to you from the fish changes direction when it leaves the tank and, in this case, light rays traveling along two different paths both reach our eyes. The changing of a light ray's direction (loosely called *bending*) when it passes a boundary between materials of different composition, or between layers in single material where there are changes in temperature and density, is called *refraction*. Refraction is responsible for a tremendous range of optical phenomena, from the action of lenses to voice transmission through optical fibers.

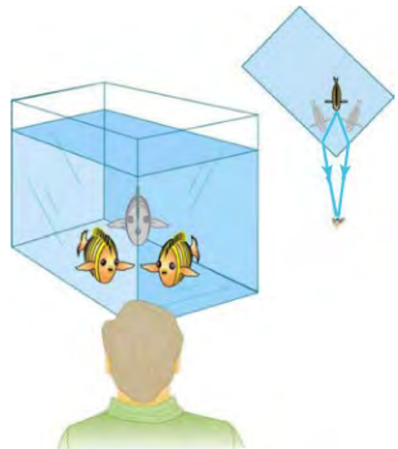

Figure 16.16 Looking at the fish tank as shown, we can see the same fish in two different locations, because light changes directions when it passes from water to air. In this case, light rays traveling on two different paths change direction as they travel from water to air, and so reach the observer. Consequently, the fish appears to be in two different places. This bending of light is called *refraction* and is responsible for many optical phenomena.

Why does light change direction when passing from one material (medium) to another? It is because light changes speed when going from one material to another. This behavior is typical of all waves and is especially easy to apply to light because light waves have very small wavelengths, and so they can be treated as rays. Before we study the law of refraction, it is useful to discuss the speed of light and how it varies between different media.

The speed of light is now known to great precision. In fact, the speed of light in a vacuum, c, is so important, and is so precisely known, that it is accepted as one of the basic physical quantities, and has the fixed value

$$c = 2.9972458 \times 10^8 \text{m/s} \approx 3.00 \times 10^8 \text{m/s} \quad (16.4)$$

where the approximate value of 3.00×10^8 m/s is used whenever three-digit precision is sufficient. The speed of light through matter is less than it is in a vacuum, because light interacts with atoms in a material. The speed of light depends strongly on the type of material, given that its interaction with different atoms, crystal lattices, and other substructures varies. We define the **index of refraction**, n, of a material to be

$$n = \frac{c}{v},$$

where v is the observed speed of light in the material. Because the speed of light is always less than c in matter and equals c only in a vacuum, the index of refraction (plural: indices of refraction) is always greater than or equal to one.

Table 16.2 lists the indices of refraction in various common materials.

Medium	n
Gases at 0 °C and 1 atm	

Table 16.2 Indices of Refraction The table lists the indices of refraction for various materials that are transparent to light. Note, that light travels the slowest in the materials with the greatest indices of refraction.

Medium	n
Air	1.000293
Carbon dioxide	1.00045
Hydrogen	1.000139
Oxygen	1.000271
Liquids at 20 °C	
Benzene	1.501
Carbon disulfide	1.628
Carbon tetrachloride	1.461
Ethanol	1.361
Glycerin	1.473
Water, fresh	1.333
Solids at 20 °C	
Diamond	2.419
Fluorite	1.434
Glass, crown	1.52
Glass, flint	1.66
Ice at 0 °C	1.309
Plexiglas	1.51
Polystyrene	1.49
Quartz, crystalline	1.544
Quartz, fused	1.458
Sodium chloride	1.544

Table 16.2 Indices of Refraction The table lists the indices of refraction for various materials that are transparent to light. Note, that light travels the slowest in the materials with the greatest indices of refraction.

Medium	n
Zircon	1.923

Table 16.2 Indices of Refraction The table lists the indices of refraction for various materials that are transparent to light. Note, that light travels the slowest in the materials with the greatest indices of refraction.

Figure 16.17 provides an analogy for and a description of how a ray of light changes direction when it passes from one medium to another. As in the previous section, the angles are measured relative to a perpendicular to the surface at the point where the light ray crosses it. The change in direction of the light ray depends on how the speed of light changes. The change in the speed of light is related to the indices of refraction of the media involved. In the situations shown in Figure 16.17, medium 2 has a greater index of refraction than medium 1. This difference in index of refraction means that the speed of light is less in medium 2 than in medium 1. Note that, in Figure 16.17(a), the path of the ray moves closer to the perpendicular when the ray slows down. Conversely, in Figure 16.17(b), the path of the ray moves away from the perpendicular when the ray speeds up. The path is exactly reversible. In both cases, you can imagine what happens by thinking about pushing a lawn mower from a footpath onto grass, and vice versa. Going from the footpath to grass, the right front wheel is slowed and pulled to the side as shown. This is the same change in direction for light when it goes from a fast medium to a slow one. When going from the grass to the footpath, the left front wheel moves faster than the others, and the mower changes direction as shown. This, too, is the same change in direction as light going from slow to fast.

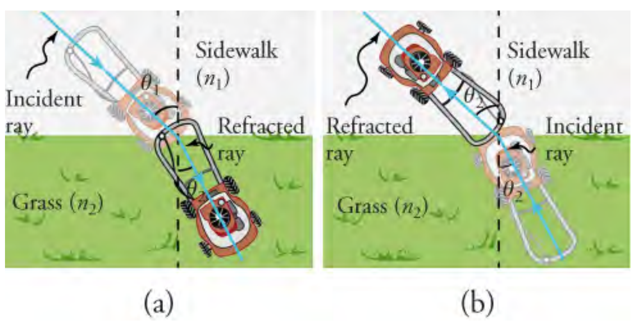

Figure 16.17 The change in direction of a light ray depends on how the speed of light changes when it crosses from one medium to another. For the situations shown here, the speed of light is greater in medium 1 than in medium 2. (a) A ray of light moves closer to the perpendicular when it slows down. This is analogous to what happens when a lawnmower goes from a footpath (medium 1) to grass (medium 2). (b) A ray of light moves away from the perpendicular when it speeds up. This is analogous to what happens when a lawnmower goes from grass (medium 2) to the footpath (medium 1). The paths are exactly reversible.

Snap Lab

Bent Pencil

A classic observation of refraction occurs when a pencil is placed in a glass filled halfway with water. Do this and observe the shape of the pencil when you look at it sideways through air, glass, and water.

- A full-length pencil
- A glass half full of water

Instructions

Procedure

1. Place the pencil in the glass of water.
2. Observe the pencil from the side.

3. Explain your observations.

Virtual Physics

Bending Light

Click to view content (https://www.openstax.org/l/28Bendinglight)

The Bending Light simulation in allows you to show light refracting as it crosses the boundaries between various media (download animation first to view). It also shows the reflected ray. You can move the protractor to the point where the light meets the boundary and measure the angle of incidence, the **angle of refraction**, and the angle of reflection. You can also insert a prism into the beam to view the spreading, or **dispersion**, of white light into colors, as discussed later in this section. Use the ray option at the upper left.

A light ray moving upward strikes a horizontal boundary at an acute angle relative to the perpendicular and enters the medium above the boundary. What must be true for the light to bend away from the perpendicular?

a. The medium below the boundary must have a greater index of refraction than the medium above.
b. The medium below the boundary must have a lower index of refraction than the medium above.
c. The medium below the boundary must have an index of refraction of zero.
d. The medium above the boundary must have an infinite index of refraction.

The amount that a light ray changes direction depends both on the incident angle and the amount that the speed changes. For a ray at a given incident angle, a large change in speed causes a large change in direction, and thus a large change in the angle of refraction. The exact mathematical relationship is the law of refraction, or ***Snell's law***, which is stated in equation form as

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \text{ or } \frac{n_1}{n_2} = \frac{\sin\theta_2}{\sin\theta_1}.$$

In terms of speeds, Snell's law becomes

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2}. \quad (16.5)$$

Here, n_1 and n_2 are the indices of refraction for media 1 and 2, respectively, and θ_1 and θ_2 are the angles between the rays and the perpendicular in the respective media 1 and 2, as shown in Figure 16.17. The incoming ray is called the ***incident ray*** and the outgoing ray is called the ***refracted ray***. The associated angles are called the *angle of incidence* and the *angle of refraction*. Later, we apply Snell's law to some practical situations.

Dispersion is defined as the spreading of white light into the wavelengths of which it is composed. This happens because the index of refraction varies slightly with wavelength. Figure 16.18 shows how a prism disperses white light into the colors of the rainbow.

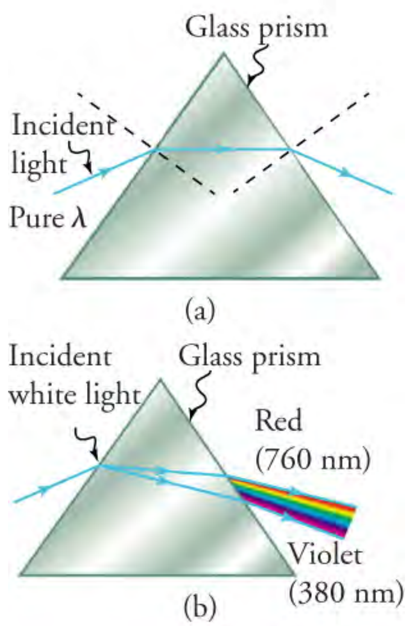

Figure 16.18 (a) A pure wavelength of light (λ) falls onto a prism and is refracted at both surfaces. (b) White light is dispersed by the prism (spread of light exaggerated). Because the index of refraction varies with wavelength, the angles of refraction vary with wavelength. A sequence of red to violet is produced, because the index of refraction increases steadily with decreasing wavelength.

Rainbows are produced by a combination of refraction and reflection. You may have noticed that you see a rainbow only when you turn your back to the Sun. Light enters a drop of water and is reflected from the back of the drop, as shown in Figure 16.19. The light is refracted both as it enters and as it leaves the drop. Because the index of refraction of water varies with wavelength, the light is dispersed and a rainbow is observed.

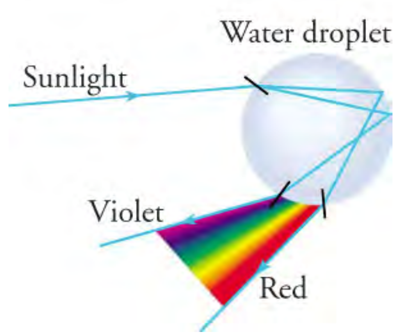

Figure 16.19 Part of the light falling on this water drop enters and is reflected from the back of the drop. This light is refracted and dispersed both as it enters and as it leaves the drop.

WATCH PHYSICS

Dispersion

This video explains how refraction disperses white light into its composite colors.

Click to view content (https://www.openstax.org/l/28Raindrop)

Which colors of the rainbow bend most when refracted?

a. Colors with a longer wavelength and higher frequency bend most when refracted.
b. Colors with a shorter wavelength and higher frequency bend most when refracted.
c. Colors with a shorter wavelength and lower frequency bend most when refracted.
d. Colors with a longer wavelength and a lower frequency bend most when refracted.

A good-quality mirror reflects more than 90 percent of the light that falls on it; the mirror absorbs the rest. But, it would be useful to have a mirror that reflects all the light that falls on it. Interestingly, we can produce total reflection using an aspect of refraction. Consider what happens when a ray of light strikes the surface between two materials, such as is shown in Figure 16.20(a). Part of the light crosses the boundary and is refracted; the rest is reflected. If, as shown in the figure, the index of refraction for the second medium is less than the first, the ray bends away from the perpendicular. Because $n_1 > n_2$, the angle of refraction is greater than the angle of incidence—that is, $\theta_2 > \theta_1$. Now, imagine what happens as the incident angle is increased. This causes θ_2 to increase as well. The largest the angle of refraction, θ_2 , can be is 90°, as shown in Figure 16.20(b). The **critical angle**, θ_c , for a combination of two materials is defined to be the incident angle, θ_1 , which produces an angle of refraction of 90°. That is, θ_c is the incident angle for which $\theta_2 = 90°$. If the incident angle, θ_1 , is greater than the critical angle, as shown in Figure 16.20(c), then all the light is reflected back into medium 1, a condition called ***total internal reflection***.

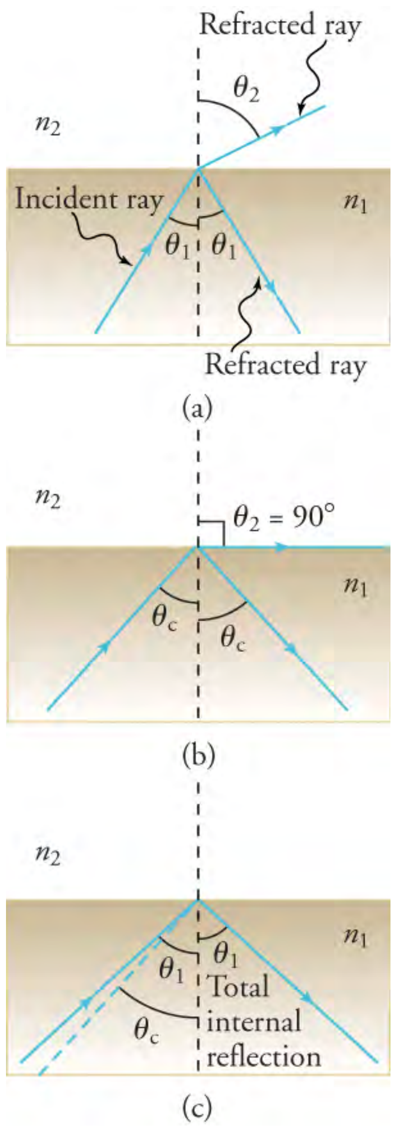

Figure 16.20 (a) A ray of light crosses a boundary where the speed of light increases and the index of refraction decreases—that is, $n_2 < n_1$. The refracted ray bends away from the perpendicular. (b) The critical angle, θ_c, is the one for which the angle of refraction is 90°. (c) Total internal reflection occurs when the incident angle is greater than the critical angle.

Recall that Snell's law states the relationship between angles and indices of refraction. It is given by

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 . \tag{16.6}$$

When the incident angle equals the critical angle ($\theta_1 = \theta_c$), the angle of refraction is 90° ($\theta_2 = 90°$). Noting that sin 90° = 1, Snell's law in this case becomes

$$n_1 \sin\theta_1 = n_2.$$

The critical angle, θ_c, for a given combination of materials is thus

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right),$$

for $n_1 > n_2$.

Total internal reflection occurs for any incident angle greater than the critical angle, θ_c, and it can only occur when the second medium has an index of refraction less than the first. Note that the previous equation is written for a light ray that travels in medium 1 and reflects from medium 2, as shown in Figure 16.20.

There are several important applications of total internal reflection. Total internal reflection, coupled with a large index of refraction, explains why diamonds sparkle more than other materials. The critical angle for a diamond-to-air surface is only 24.4°; so, when light enters a diamond, it has trouble getting back out (Figure 16.21). Although light freely enters the diamond at different angles, it can exit only if it makes an angle less than 24.4° with the normal to a given surface. Facets on diamonds are specifically intended to make this unlikely, so that the light can exit only in certain places. Diamonds with very few impurities are very clear, so the light makes many internal reflections and is concentrated at the few places it can exit—hence the sparkle.

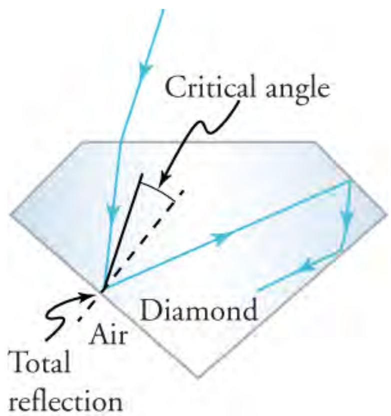

Figure 16.21 Light cannot escape a diamond easily because its critical angle with air is so small. Most reflections are total and the facets are placed so that light can exit only in particular ways, thus concentrating the light and making the diamond sparkle.

A light ray that strikes an object that consists of two mutually perpendicular reflecting surfaces is reflected back exactly parallel to the direction from which it came. This parallel reflection is true whenever the reflecting surfaces are perpendicular, and it is independent of the angle of incidence. Such an object is called a ***corner reflector*** because the light bounces from its inside corner. Many inexpensive reflector buttons on bicycles, cars, and warning signs have corner reflectors designed to return light in the direction from which it originates. Corner reflectors are perfectly efficient when the conditions for total internal reflection are satisfied. With common materials, it is easy to obtain a critical angle that is less than 45°. One use of these perfect *mirrors* is in binoculars, as shown in Figure 16.22. Another application is for periscopes used in submarines.

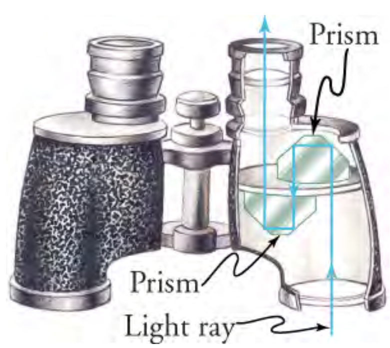

Figure 16.22 These binoculars use corner reflectors with total internal reflection to get light to the observer's eyes.

Fiber optics are one common application of total internal reflection. In communications, fiber optics are used to transmit telephone, internet, and cable TV signals, and they use the transmission of light down fibers of plastic or glass. Because the fibers are thin, light entering one is likely to strike the inside surface at an angle greater than the critical angle and, thus, be totally reflected (Figure 16.23). The index of refraction outside the fiber must be smaller than inside, a condition that is satisfied easily by coating the outside of the fiber with a material that has an appropriate refractive index. In fact, most fibers have a varying refractive index to allow more light to be guided along the fiber through total internal reflection. Rays are reflected around corners as shown in the figure, making the fibers into tiny *light pipes*.

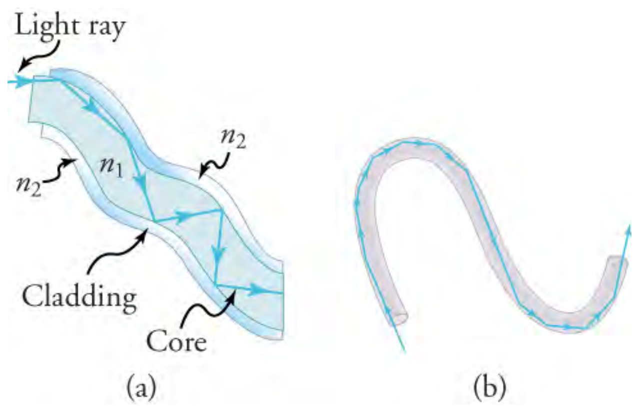

Figure 16.23 (a) Fibers in bundles are clad by a material that has a lower index of refraction than the core to ensure total internal reflection, even when fibers are in contact with one another. A single fiber with its cladding is shown. (b) Light entering a thin fiber may strike the inside surface at large, or grazing, angles, and is completely reflected if these angles exceed the critical angle. Such rays continue down the fiber, even following it around corners, because the angles of reflection and incidence remain large.

LINKS TO PHYSICS

Medicine: Endoscopes

A medical device called an *endoscope* is shown in Figure 16.24.

Figure 16.24 Endoscopes, such as the one drawn here, send light down a flexible fiber optic tube, which sends images back to a doctor in charge of performing a medical procedure.

The word *endoscope means looking inside*. Doctors use endoscopes to look inside hollow organs in the human body and inside body cavities. These devices are used to diagnose internal physical problems. Images may be transmitted to an eyepiece or sent to a video screen. Another channel is sometimes included to allow the use of small surgical instruments. Such surgical procedures include collecting biopsies for later testing, and removing polyps and other growths.

Identify the process that allows light and images to travel through a tube that is not straight.

a. The process is refraction of light.
b. The process is dispersion of light.
c. The process is total internal reflection of light.
d. The process is polarization of light.

Calculations with the Law of Refraction

The calculation problems that follow require application of the following equations:

$$n = \frac{c}{v}, \tag{16.8}$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \text{ or } \frac{n_1}{n_2} = \frac{\sin\theta_2}{\sin\theta_1},$$

and

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right), \text{ for } n_1 > n_2.$$

These are the equations for refractive index, the mathematical statement of the law of refraction (Snell's law), and the equation for the critical angle.

WATCH PHYSICS

Snell's Law Example 1

This video leads you through calculations based on the application of the equation that represents Snell's law.

Click to view content (https://www.openstax.org/l/28Snellslaw)

Which two types of variables are included in Snell's law?

a. The two types of variables are density of a material and the angle made by the light ray with the normal.
b. The two types of variables are density of a material and the thickness of a material.
c. The two types of variables are refractive index and thickness of each material.
d. The two types of variables are refractive index of a material and the angle made by a light ray with the normal.

WORKED EXAMPLE

Calculating Index of Refraction from Speed

Calculate the index of refraction for a solid medium in which the speed of light is 2.012×10^8 m/s, and identify the most likely substance, based on the previous table of indicies of refraction.

STRATEGY

We know the speed of light, *c*, is 3.00×10^8 m/s, and we are given *v*. We can simply plug these values into the equation for index of refraction, *n*.

Solution

$$n = \frac{c}{v} = \frac{3.00 \times 10^8 \text{m/s}}{2.012 \times 10^8 \text{m/s}} = 1.49 \tag{16.9}$$

This value matches that of polystyrene exactly, according to the table of indices of refraction (Table 16.2).

Discussion

The three-digit approximation for *c* is used, which in this case is all that is needed. Many values in the table are only given to three significant figures. Note that the units for speed cancel to yield a dimensionless answer, which is correct.

WORKED EXAMPLE

Calculating Index of Refraction from Angles

Suppose you have an unknown, clear solid substance immersed in water and you wish to identify it by finding its index of refraction. You arrange to have a beam of light enter it at an angle of 45.00°, and you observe the angle of refraction to be 40.30°. What are the index of refraction of the substance and its likely identity?

STRATEGY

We must use the mathematical expression for the law of refraction to solve this problem because we are given angle data, not speed data.

$$\frac{n_1}{n_2} = \frac{\sin\theta_2}{\sin\theta_1} \tag{16.10}$$

The subscripts 1 and 2 refer to values for water and the unknown, respectively, where 1 represents the medium from which the light is coming and 2 is the new medium it is entering. We are given the angle values, and the table of indicies of refraction gives us n for water as 1.333. All we have to do before solving the problem is rearrange the equation

$$n_2 = \frac{n_1 \sin\theta_1}{\sin\theta_2}. \quad (16.11)$$

Solution

$$n_2 = \frac{(1.333)(0.7071)}{0.6468} = 1.457 \quad (16.12)$$

The best match from Table 16.2 is fused quartz, with $n = 1.458$.

Discussion

Note the relative sizes of the variables involved. For example, a larger angle has a larger sine value. This checks out for the two angles involved. Note that the smaller value of θ_2 compared with θ_1 indicates the ray has bent *toward* normal. This result is to be expected if the unknown substance has a greater n value than that of water. The result shows that this is the case.

WORKED EXAMPLE

Calculating Critical Angle

Verify that the critical angle for light going from water to air is 48.6°. (See Table 16.2, the table of indices of refraction.)

STRATEGY

First, choose the equation for critical angle

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right), \text{ for } n_1 > n_2. \quad (16.13)$$

Then, look up the n values for water, n_1, and air, n_2. Find the value of $\frac{n_2}{n_1}$. Last, find the angle that has a sine equal to this value and it compare with the given angle of 48.6°.

Solution

For water, $n_1 = 1.333$; for air, $n_2 = 1.0003$. So,

$$\frac{n_2}{n_1} = \frac{1.0003}{1.333} = 0.7504$$
$$\sin^{-1}(0.7504) = 48.63°. \quad (16.14)$$

Discussion

Remember, when we try to find a critical angle, we look for the angle at which light can no longer escape past a medium boundary by refraction. It is logical, then, to think of subscript 1 as referring to the medium the light is trying to leave, and subscript 2 as where it is trying (unsuccessfully) to go. So water is 1 and air is 2.

Practice Problems

6. The refractive index of ethanol is 1.36. What is the speed of light in ethanol?
   a. 2.25×108 m/s
   b. 2.21×107 m/s
   c. 2.25×109 m/s
   d. 2.21×108 m/s

7. The refractive index of air is 1.0003 and the refractive index of crystalline quartz is 1.544. What is the critical angle for a ray of light going from crystalline quartz into air?
   a. 49.61°
   b. 20.19°
   c. 0.6479 rad
   d. 0.7048 rad

Check Your Understanding

8. Which law is expressed by the equation $n_1 \sin \vartheta_1 = n_2 \sin \vartheta_2$?
   a. This is Ohm's law.
   b. This is Wien's displacement law.
   c. This is Snell's law.
   d. This is Newton's law.

9. Explain why the index of refraction is always greater than or equal to one.
   a. The formula for index of refraction, n, of a material is $n = \frac{\text{speed of light in a material}}{\text{speed of light in a vacuum}} = \frac{v}{c}$, where $v > c$, so n is always greater than one.
   b. The formula for index of refraction, n, of a material is $n = \frac{\text{speed of light in a vacuum}}{\text{speed of light in a material}} = \frac{c}{v}$, where $c > v$, so n is always greater than one.
   c. The formula for index of refraction, n, of a material is $n = \text{speed of light in a vacuum} \times \text{speed of light in a materaial} = c \times v$, where $c, v > 1$, so n is always greater than one.
   d. The formula for refractive index, n, of a material is $n = \frac{1}{\text{speed of light in a vacuum} \times \text{speed of light in a material}} = \frac{1}{c \times v}$, where $c < v < 1$, so n is always greater than one.

10. Write an equation that expresses the law of refraction.
   a. $\frac{n_1}{n_2} = \frac{\sin \theta_1}{\sin \theta_2}$
   b. $\frac{n_2}{n_1} = \left(\frac{\sin \theta_2}{\sin \theta_1}\right)^2$
   c. $\frac{n_1}{n_2} = \left(\frac{\sin \theta_2}{\sin \theta_1}\right)^2$
   d. $\frac{n_1}{n_2} = \frac{\sin \theta_2}{\sin \theta_1}$

16.3 Lenses

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe and predict image formation and magnification as a consequence of refraction through convex and concave lenses, use ray diagrams to confirm image formation, and discuss how these properties of lenses determine their applications
- Explain how the human eye works in terms of geometric optics
- Perform calculations, based on the thin-lens equation, to determine image and object distances, focal length, and image magnification, and use these calculations to confirm values determined from ray diagrams

Section Key Terms

aberration	chromatic aberration	concave lens	converging lens	convex lens
diverging lens	eyepiece	objective	ocular	parfocal

Characteristics of Lenses

Lenses are found in a huge array of optical instruments, ranging from a simple magnifying glass to the eye to a camera's zoom lens. In this section, we use the law of refraction to explore the properties of lenses and how they form images.

Some of what we learned in the earlier discussion of curved mirrors also applies to the study of lenses. Concave, convex, focal point F, and focal length *f* have the same meanings as before, except each measurement is made from the center of the lens instead of the surface of the mirror. The **convex lens** shown in Figure 16.25 has been shaped so that all light rays that enter it parallel to its central axis cross one another at a single point on the opposite side of the lens. The central axis, or axis, is defined to be a line normal to the lens at its center. Such a lens is called a **converging lens** because of the converging effect it has on light rays. An expanded view of the path of one ray through the lens is shown in Figure 16.25 to illustrate how the ray changes